Networks with Dynamic Topologies: A Greedy Algorithm for Satellite Placement Problem

Umut Topkara, İlhan Uysal and İlker Yoncacı
Department of Computer Engineering, Bilkent University
Ankara, 06533, Turkey
{topkara,uilhan,yoncaci}@cs.bilkent.edu.tr

Abstract

Satellite placement problem for dynamic communication network topologies is presented and a solution with a greedy algorithm is described. The problem to be solved is to provide continuous communication connection between terminal nodes in a dynamic environment. The model used in this paper is the solar system where satellites and planets orbit around the sun. For simplification all orbits are assumed to be circular. Satellites and planets contain transmission stations that have a constant transmission range, and both are used to establish communication. The greedy algorithm finds small number of satellite locations for this model to maintain continuity of communication between planets.

Keywords: Dynamic networks, satellite placement, concentrator location.

1 Introduction

An important problem in communication networks is to establish continuity of communication between terminal nodes by employing intermediate nodes or switches. Generally for conventional networks, fixed concentrator locations are determined for stationary terminal nodes[2, 3, 5]. In wireless networks mobility of wireless terminals is established by using stationary intermediate nodes [1]. In this paper we present a new concentrator location problem where both terminal and intermediate nodes are mobile.

In this problem the motions of intermediate and terminal nodes are constrained. In order to configure such a dynamic environment we consider the solar system as a model. In this configuration satellites and planets contain transmission stations that have a constant transmission range, and these stations are used to establish communication. Our greedy algorithm finds the appropriate locations of a small number of intermediate nodes (i.e. satellites) for this model to maintain continuity of communication between terminal nodes (i.e. planets).

The motion of terminal nodes and intermediate nodes are constrained to circular paths around the sun, and their angular velocities are determined by the radii of their orbits around the sun. For a given distance (radius) of an object (planet or satellite) from the sun, the orbit and the average angular velocity (w) is determined by using Newton's law of gravitational force[6]:

$$F = G\left[\frac{m_1 m_2}{r^2}\right]$$

Note that the angular velocity of the objects are not linearly related to radius of their orbits. As a result of this, the topology of the outer network will change by time due to relatively large angular velocity of inner planets.

In order to construct a simple model and to be able to determine a solution for the problem, some simplifications and assumptions are made. For example we use circular orbits instead of elliptical ones in order to determine distances of orbiting objects easily. Such assumptions are described in the following sections in detail.

The solution we propose is to use a greedy algorithm to determine satellite locations, their distances from the sun and their relative angular positions. The Greedy Satellite Location (GSL) algorithm we developed, makes a greedy search among many possible satellite positions. By pruning all these possible satellite positions, we try to reach a minimum set of satellites that establish communication between planets for all possible positions of them.

In the next section, we describe the problem formally and give physical and mathematical background related with the problem. The simplifications and assumptions are also described in this section. In Section 3, we describe the GSL algorithm, and experimental results are presented Section 4.

2 Formal Specifications

In this section we describe formulations, problem description, assumptions and simplifications for the satellite placement problem.

2.1 Physical and Mathematical Formulations

Kepler's Third Law [4] states that the period P of revolution of a planet is directly proportional to its mean distance from the sun:

$$P^2 = Ka^3$$

The constant K in the equation has been shown by Newton to be

$$K = \frac{4\pi^2}{[G(M_{sun} + M_{planet})]}$$

where G is gravitational constant and M_{sun} and M_{planet} are mass of the sun and mass of the planet respectively. Since mass of the planet is very small compared to the mass of the sun, it is possible to treat K as constant in calculations for the objects rotating around the sun in our solar system.

It is possible to derive a relation between the angular velocity and mean distance from the sun using the equation:

$$\left(\frac{2\pi}{w}\right)^2 = Ka^3$$

$$w = 2\pi \left[\frac{1}{Ka^3}\right]^{1/2}$$

The above relation can be used to find the relative location of a planet or any object with respect to the sun after a period of time, if the initial position is known. Moreover it is possible to calculate distance of a planet to another if their initial positions are known. Let us describe the positions of the objects in polar coordinates and $O_1(\theta_1, r_1)$ and $O_2(\theta_2, r_2)$ be positions of these objects. θ_1 and θ_2 are angular positions for two different objects and r_1 and r_2 are their distance from the sun respectively. If we simplify our model by letting the planets rotate in circular orbits rather than elliptical:

$$w_1 = 2\pi \left[\frac{1}{Kr_1^3}\right]^{1/2}$$

$$w_2 = 2\pi \left[\frac{1}{Kr_2^3}\right]^{1/2}$$

$$D_{1,2,t=0} = [r_1^2 + r_2^2 - 2cos(\theta_1 - \theta_2)r_1r_2]^{1/2}$$

where w_1 and w_2 are angular velocities and $D_{1,2,t}$ is the distance between objects O_1 and O_2 rotating around sun. Orbits of objects are assumed to be circular. The distance at any time t is computed by using the following formula.

$$D_{1,2,t} = [r_1^2 + r_2^2 - 2cos([\theta_1 + tw_1] - [\theta_2 + tw_2])r_1r_2]^{1/2} \qquad (1)$$

2.2 Satellite Location Problem

In a planetary system, planets rotate around a sun. Each planet has an elliptical orbit and the angular velocity of the planet is inversely proportional to the cube of the square root of its average distance from the sun. Since angular velocity of planets are different; the relative positions of planets in the system vary by time.

Consider a communication network consisting of terminal nodes located on the planets and intermediate nodes which are satellites orbiting around the sun. The transmission station on each planet has some coverage area and the satellites also have the same coverage area for their transmission.

One of the problems associated with such a network is to keep all terminal nodes in the network connected. Since the terminal nodes and intermediate nodes do not have stationary relative locations, satellites should be located so that, connection between planets should be preserved at all times with the minimum number of satellites.

2.3 Assumptions and Simplifications

It has already been stated that the planets travel on circular orbits rather than elliptical. In our model, the sizes and masses of objects (planets and satellites) are neglected. The gravitational forces among satellites and planets are also ignored. We also assume that all planets have different orbits. This is also the actual case for solar system. However many satellites can be placed on orbits including the orbits of planets. Furthermore, it will be assumed that the orbits of the planets lie on the same plane.

The topology of the network described above is completely defined for a given time instance, furthermore it is possible to determine the time of connection establishments for any two objects in the system. However solving the problem using direct calculation as continuous problem is not possible since there is a decision making is involved among infinitely many choices i.e. possible location of a satellite can be any point on the plane of the orbits of the planets.

To convert the problem from continuous domain to discrete domain several further assumptions are made. The satellites are located on orbits that have radii as integer multiples of a fraction of the coverage area radii R of satellites. Possible positions on each orbit are points that divide the circle of the orbit to a number of unit length arcs, which is a fraction of R. For further simplicity it is assumed that satellites and transmission stations on planets have same coverage area radii R.

3 Greedy Satellite Location Algorithm

There are two possible approaches for the satellite placement problem. First approach is to constructively place the satellites on orbits, and the second approach is to prune unnecessary satellites from a configuration where all possible locations for satellites are filled. We have chosen the second approach to solve the satellite placement problem.

While pruning the satellites from a configuration that has redundant satellites, we first determine the satellite that is most probable to be unnecessary, and then check whether the connectivity is still preserved without it. We have developed a greedy algorithm to prune satellites using this technique.

The greedy satellite placement algorithm uses greedy approaches. The first one selects a satellite to be removed from a given set of objects around the sun, as described above. The second one determines successive steps such that at each step a solution can be found for a given set of rotating objects. The first greedy approach is used to be able to reach a minimum set of satellites, while the second one is used to decrease the problem size and to increase computational efficiency.

3.1 Greedy Choice for Pruning

The idea behind the greedy algorithm for pruning is that a satellite is less needed when it is too close to other satellites during its course of orbiting around the sun. The other satellites that are close to it have the possibility of rendering it redundant. And that probability is maximum when the satellite is too close to other satellites. A greedy choice here determines the satellites to be pruned using the distances of orbiting objects among each other.

In the course of movement, satellites come close to and depart from each other. In the greedy algorithm, for a period which is large enough to establish all possible relative positions of objects, we select a satellite and remove it from the topology.

In order to accomplish this reduction in a period, we check the distances of all objects (at discrete time intervals). The number of distance computations is proportional to the square of the number of objects. At each discrete time step, each satellite determine the nearest object and records its distance. For all time steps, the maximum among these distances is found by using Eq. (1). This maximum distance is defined as $Max_t Min_j(D_{i,j,t})$, where t is the time step and j is the nearest object to any satellite i at time t. The satellite i having the minimum $Max_t Min_j(D_{i,j,t})$ is determined for the pruning in a duration period.

Intuitively, we search for the satellite that is closest to others in its worst position during its motion. The worst position here the largest distance of a satellite to its closest neighbor at any discrete time, during its rotation in the period described above. Formally, the greedy selection criterion to select a satellite i for a period can be defined as:

$$Min_i \; Max_t \; Min_j(D_{i,j,t})$$

3.2 Greedy Choice for Problem Size

The complexity of one greedy pruning computation is $O(n^2T)$ where n is the number of satellite locations and T is the number of time steps. This computation cost is too large for computing the locations of satellites even for a small number of planets that are closely placed. It is possible to introduce another greedy choice to restrict the application domain of the greedy choice for pruning. If the inner planet of two neighboring planets can communicate with the outer planet through communication satellites, establishing communication between the outer planet and the next outer planet is enough to establish communication between the inner planet and the next outer planet. This greedy choice will reduce the size of the problem. Suppose N is the size of all objects and N_i is the number objects between the orbits of two successive planets, and p is the number of planets

$$N = \sum_{i=1}^{p-1} N_i$$

The use of greedy pruning for an object size of N will require N^2 distance computations at each time step. However the use of greedy choice for efficiency will require much smaller distance computations. This is given in the following equation and it is much smaller than N^2.

$$\sum_{i=1}^{p-1} N_i^2 \quad << \quad N^2$$

By using this greedy approach to reduce computation cost, we start from the inner-most planet, and process successively to outer planet. At each step, we establish connection between two successive planets. At the termination of the algorithm, communication between all planets is established.

3.3 GSL Algorithm

We establish the above approaches iteratively and reach a final set of satellites at the end. The overview of the GSL algorithm is given in Figure 1. By using

```
Determine new orbits between planets with equal internals I < R/2
Locate satellites to each orbit with equal intervals I
For i = 1 to p − 1
   For each time duration T
      Prune a satellite between planet orbits i and i + 1
      Check connection between planets i and i + 1 using BFS
      If connection is broken relocate last pruned satellite
         and quit time duration loop
   end for
end for
```

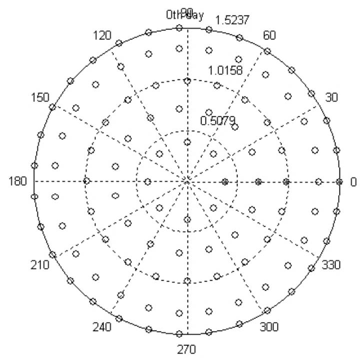
Figure 1: Greedy Satellite Location Algorithm. In order to check connection between planets we used Breadth First Search (BFS) algorithm.

the second greedy approach we decrease the computational complexity at the cost of obtaining solutions with large number of satellites when compared to the optimum solution. However, this trade off can be avoided by making a final pruning by using the greedy pruning algorithm with all remaining objects, and by checking the connectivity between all planets.

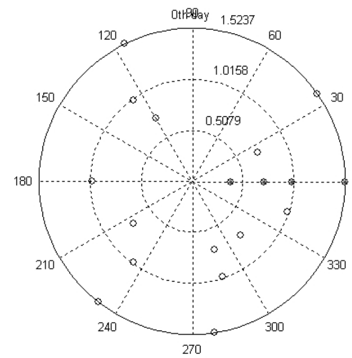
4 Experimental Results

The output of the program with the real distances of first 4 planets of our solar system are shown in Table 4 in polar coordinates. The algorithm is run on two different initial configurations. In the first case planets are placed on the same line(Table 4a), and in the second case consecutive planets are placed at 90 degrees to each other(Table 4d). In Table 4 **(b)** is the solution to first case and **(e)** is the solution to the second case. **(c)** and **(f)** show the is the orientation of objects in solutions to two cases indays 360, 720, 1080 and 1440. The unit distance used is the distance of earth from the sun. The transmission range R is taken as 1 unit. As unit interval $I = \frac{R}{\pi}$ is used. Planets are shown with black dots and satellites are shown with empty dots in those figures. The solution is shown in Table 2. In experiments, we rotated the topology for 100 years by checking the connection, and the connection was not broken among planets.

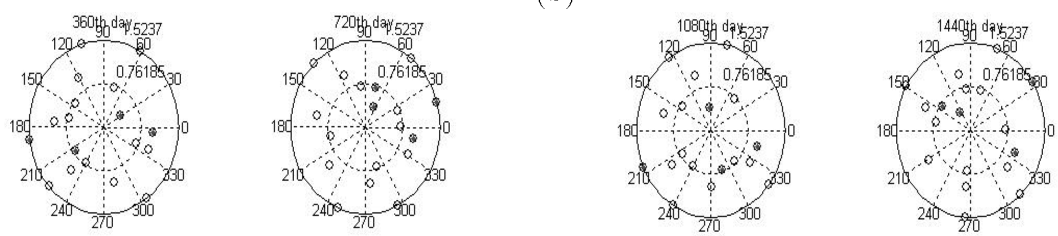
Table 1: The solution found to establish communication for the first four planets (inner planets) of our solar system. Black circles represent planets and empty circles represent satellites. There are 14 satellites in both solutions to the problems (a) and (d).



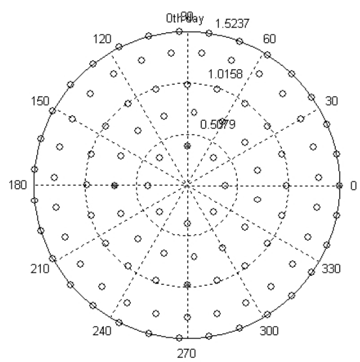
(a)



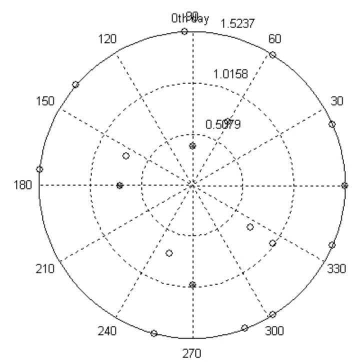
(b)



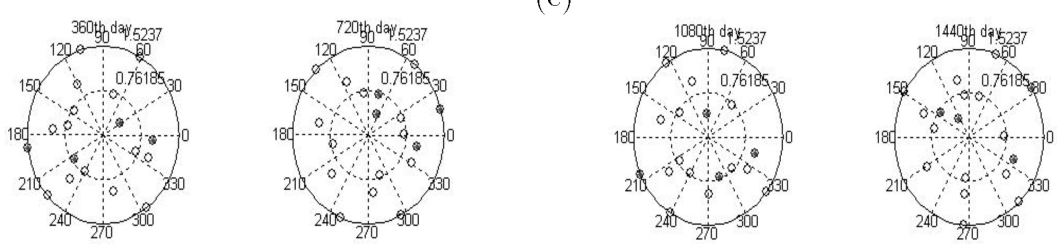
(c)



(d)



(e)



(f)

Solution to Table 4a				
Id-number	Angle(rad)	Radius	W (unit/sec)	Is-planet
1	0.00000000	0.3871	8.30E-07	1
9	0.00000000	0.7233	3.20E-07	1
10	0.41887902	0.7233	3.20E-07	0
14	2.09439510	0.7233	3.20E-07	0
18	3.76991118	0.7233	3.20E-07	0
21	5.02654825	0.7233	3.20E-07	0
22	5.44542727	0.7233	3.20E-07	0
24	0.00000000	1.0000	2.00E-07	0
31	2.19911486	1.0000	2.00E-07	0
34	3.14159265	1.0000	2.00E-07	0
37	4.08407045	1.0000	2.00E-07	0
40	5.02654825	1.0000	2.00E-07	0
43	5.96902604	1.0000	2.00E-07	0
71	0.00000000	1.5237	1.10E-07	1
74	0.60805019	1.5237	1.10E-07	0
81	2.02683397	1.5237	1.10E-07	0
91	4.05366794	1.5237	1.10E-07	0
95	4.86440153	1.5237	1.10E-07	0
Solution to Table 4d				
Id-number	Angle(rad)	Radius	W (unit/sec)	Is-planet
1	1.57079633	0.3871	8.30E-07	1
9	3.14159265	0.7233	3.20E-07	1
12	4.39822972	0.7233	3.20E-07	0
15	5.65486678	0.7233	3.20E-07	0
19	7.33038286	0.7233	3.20E-07	0
23	9.00589894	0.7233	3.20E-07	0
24	4.71238898	1.0000	2.00E-07	1
27	5.65486678	1.0000	2.00E-07	0
71	6.28318531	1.5237	1.10E-07	1
73	6.68855210	1.5237	1.10E-07	0
76	7.29660229	1.5237	1.10E-07	0
79	7.90465248	1.5237	1.10E-07	0
83	8.71538607	1.5237	1.10E-07	0
86	9.32343626	1.5237	1.10E-07	0
93	10.74222004	1.5237	1.10E-07	0
96	11.35027023	1.5237	1.10E-07	0
97	11.55295363	1.5237	1.10E-07	0
100	12.16100382	1.5237	1.10E-07	0

Table 2: Positions of satellites and planets in solutions to problems in Table 4a and Table 4d. (1 unit is earth distance to sun)

5 Conclusion

In this paper, satellite placement problem for dynamic communication networks is presented, where both terminal and concentrator nodes are mobile. The greedy GSL algorithm that has been developed as a solution for this problem finds small number of satellite locations for the solar system, where terminal nodes are planets and concentrators are satellites. According to the experimental results, GSL algorithm is successful in obtaining a feasible solution with a small number of satellites while keeping the computational cost at reasonable levels for our model, the solar system.

References

[1] Acharya A., Li J., Rajagopalan B. and Raychaudhuri D., Mobility management in wireless ATM networks, *IEEE Communications Magazine* 35: (11) 100-109, 1997.

[2] Boorstyn R. R. and Frank H., Large-Scale Network Topological Optimization, *IEEE Transactions on Communications*, Vol. COM-25, No. 1, January 1977.

[3] Feldman E. and Lehner F. A., Warehouse Locations Under Continuous Economies of Scale, *Management Science*, Vol.12, 670-684, 1966.

[4] Kepler J. and Harmonices M., *Memoirs of the American Philosophical Society*, 209, Philadelphia, 1619.

[5] Kuchn A. A., and Hamburger M. J., A Heuristic Program for Locating Warehouses, *Management Science*, Vol. 9, 643-666, 1963.

[6] Newton I., *Philosophiae Naturalis Principia Mathematica*,London, 1687.